POSITION AND DISPLACEMENT ANALYSIS

Dr. Amer Abdualhakeem

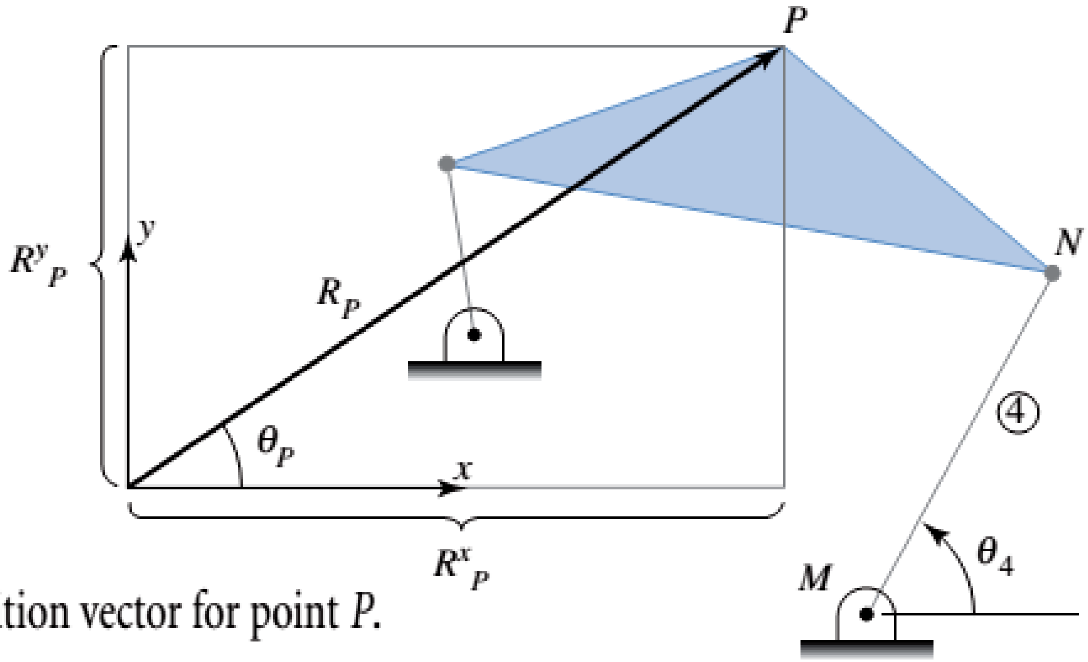
For many mechanisms, the sole(only) purpose of analysis is to determine the location of all links as the driving link(s) of the mechanism is moved into another position.

POSITION

Position refers to the location of an object. The following sections will address the position of points and links.

A-Position of a Point

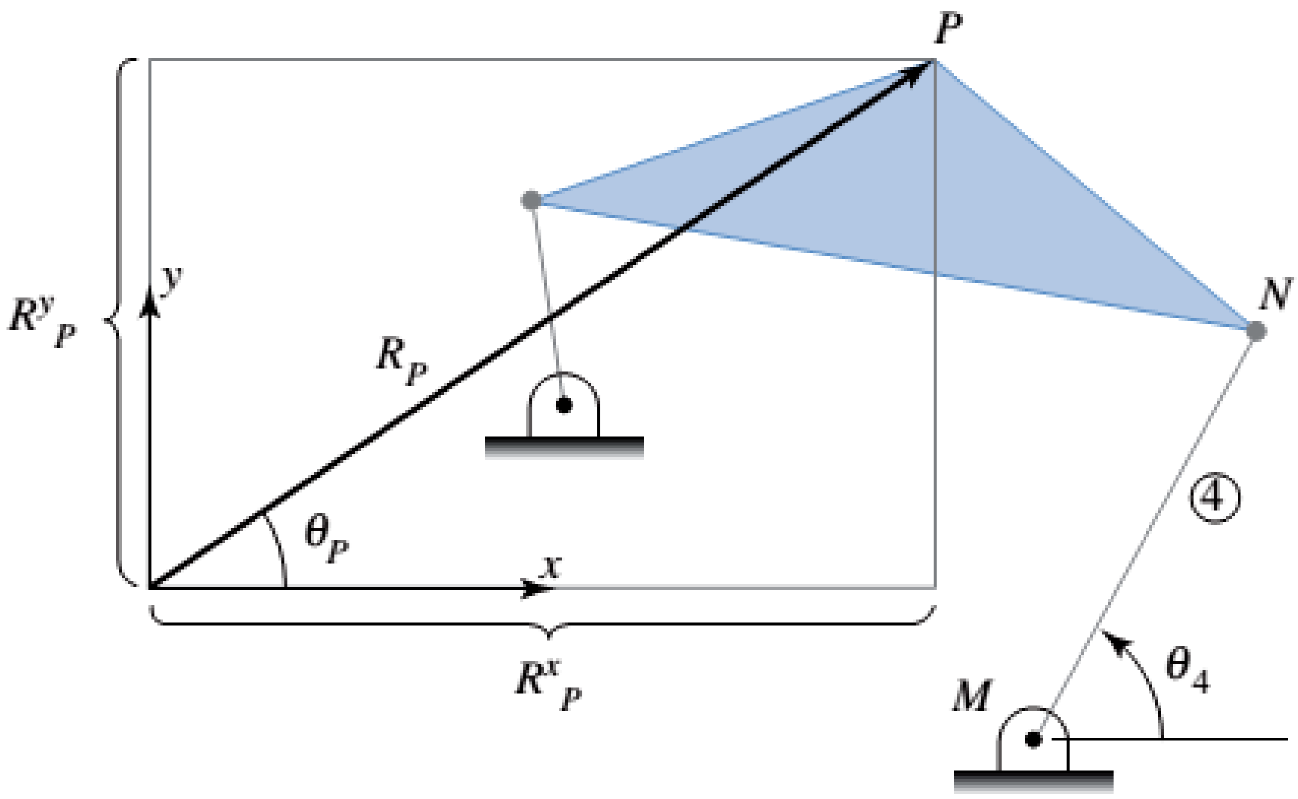
The position of a point on a mechanism is the spatial location of that point. It can be defined with a position vector, R, from a reference origin to the location of the point.



Position vector for point P.

B-Angular Position of a Link

The angular position of a link is also an important quantity. Fig, line MN lies on link 4. The angular position of link 4 is defined by , which is the angle between the -axis and line MN. For consistency, angular position is defined as positive if the angle is measured counterclockwise from the .reference



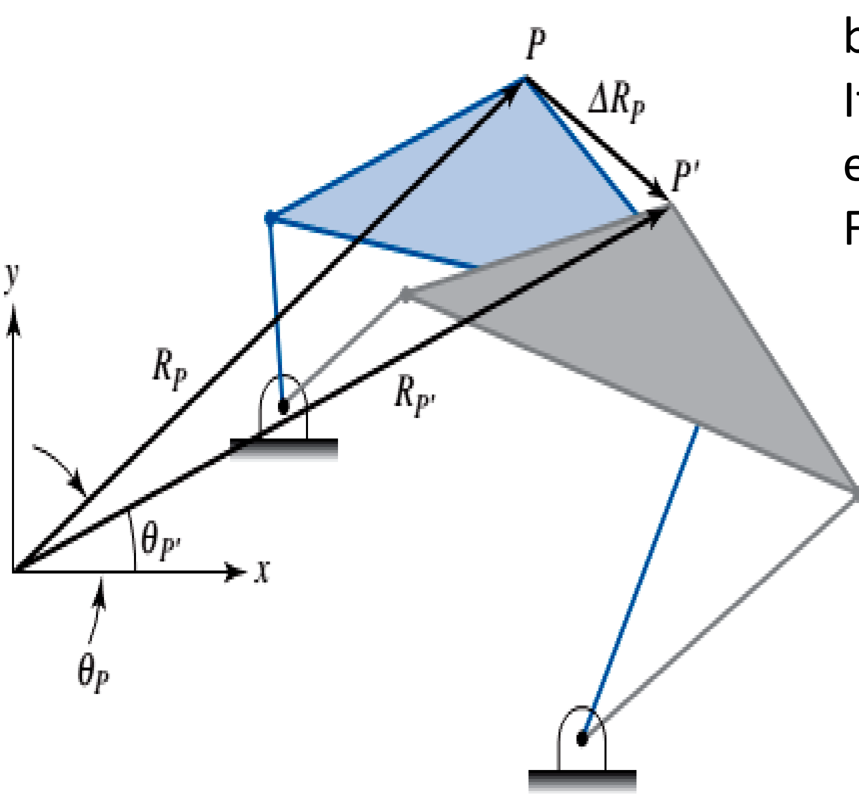
DISPLACEMENT

Displacement is the end product of motion. It is a vector that represents the distance between the starting and ending positions of a point or link. There are two types of displacements that will be considered: linear and angular.

Linear Displacement

Linear displacement, , is the straight line distance between the starting and ending position of a point during a time interval under consideration.

$$\Delta R_P = R_{P'} -> R_P$$



Angular Displacement

Angular displacement, , is the angular distance between two configurations of a rotating link. It is the difference between the starting and ending angular positions of a link, as shown in Figure 4.4.

$$\Delta\theta_3 = \theta_{3'} - \theta_3$$

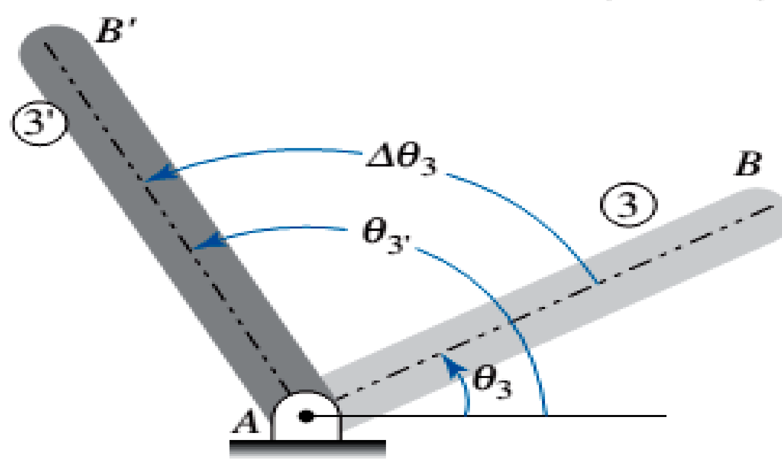


FIGURE 4.4 Angular displacement.

a specified displacement. In this example, the driving displacement is angular, $\Delta\theta_2 = 15°$clockwise.

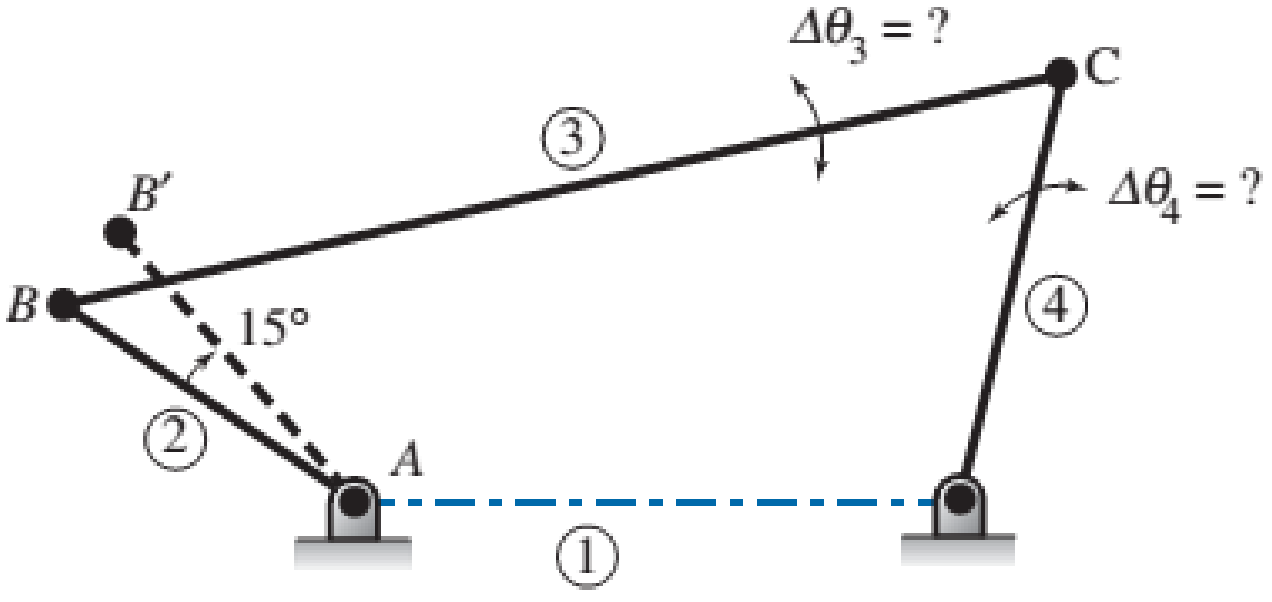
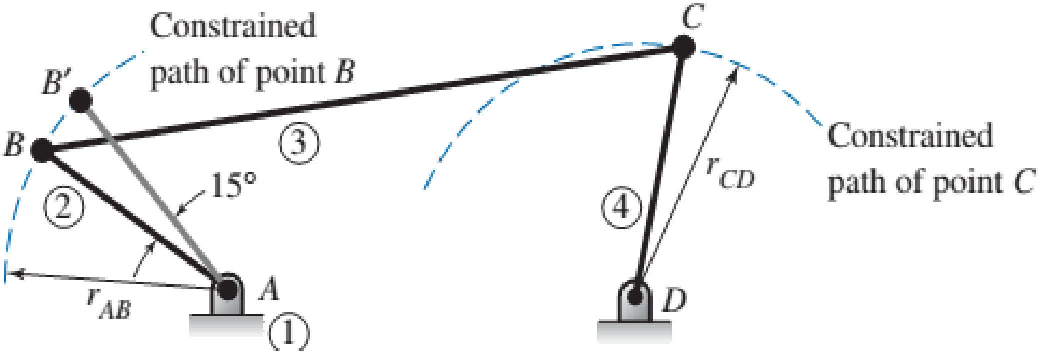
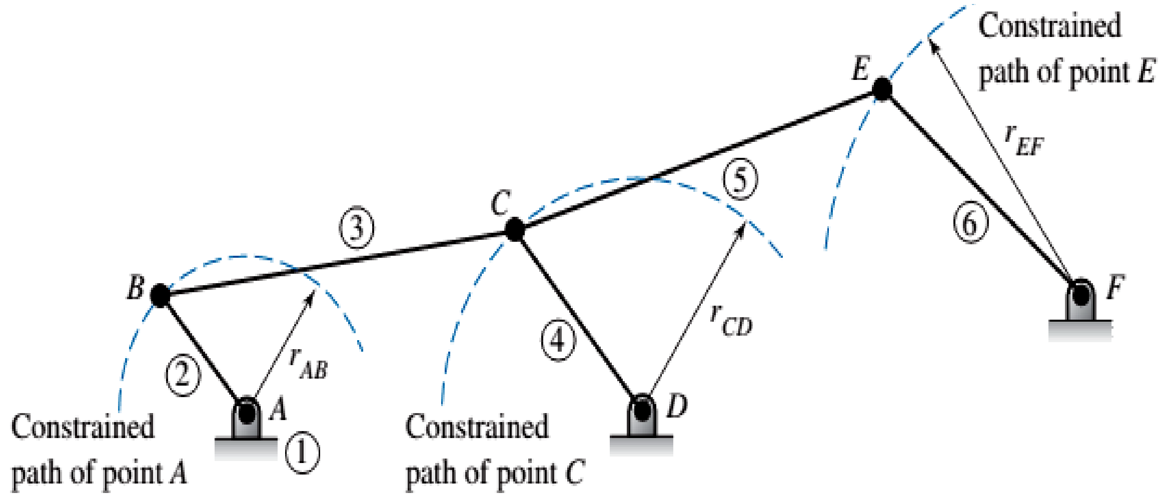
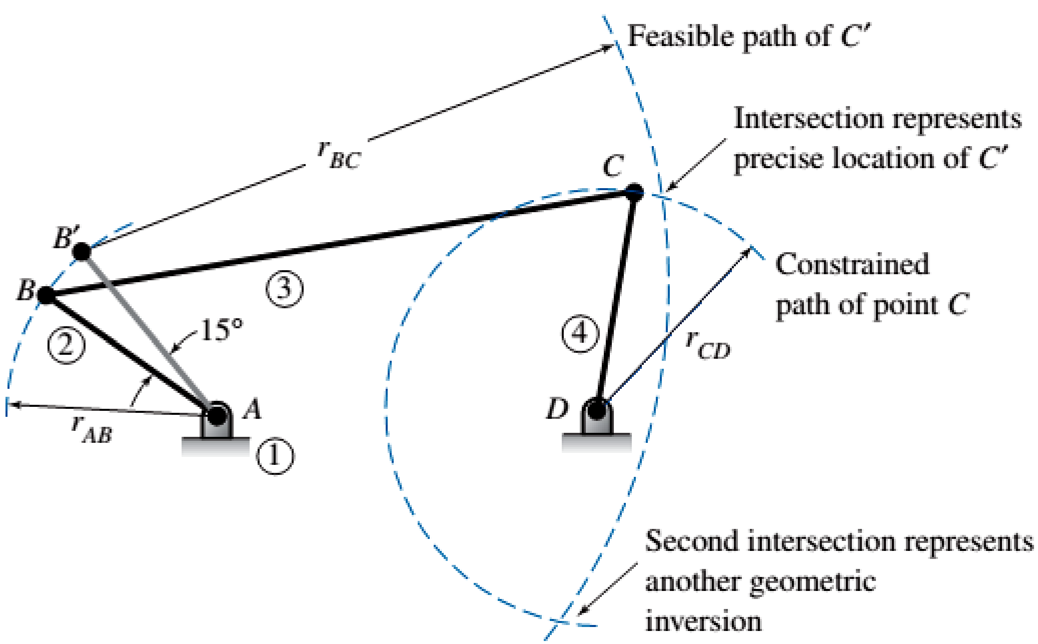


FIGURE 4.5 Typical position analysis.

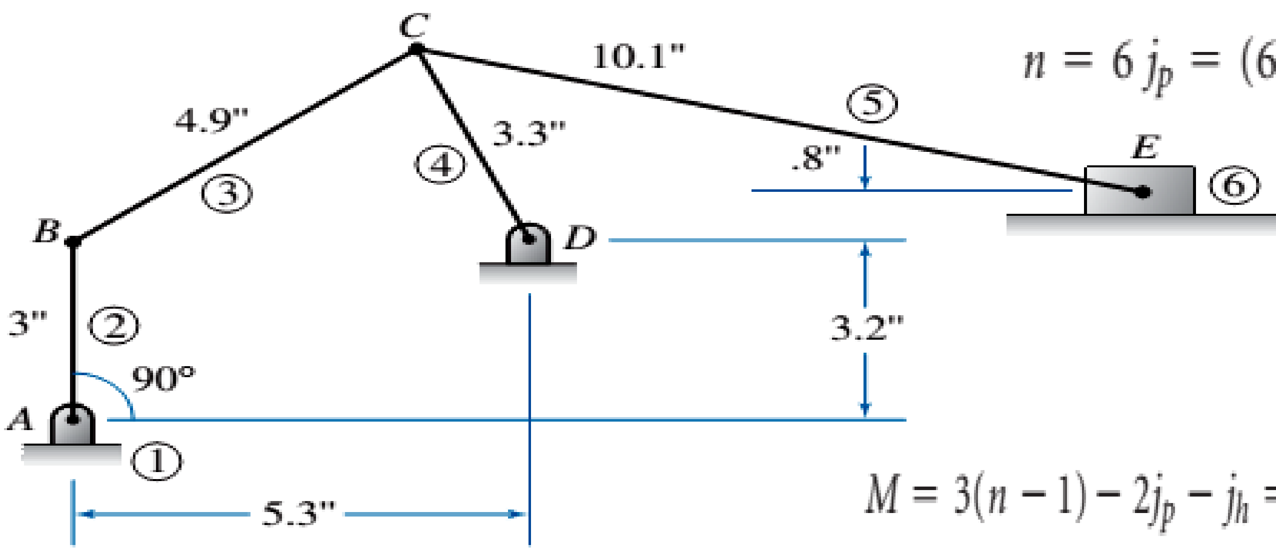
$$M = 3(4 - 1) - 2(4) = 1$$


Constrained
path of point B
C
B′
B
3
15°
2
4
r_{CD}
Constrained
path of point C
r_{AB}
A
1
D


Constrained
path of point E
E
r_{EF}
5
C
3
6
F
B
4
r_{CD}
r_{AB}
2
A
D
Constrained
path of point A
1
Constrained
path of point C

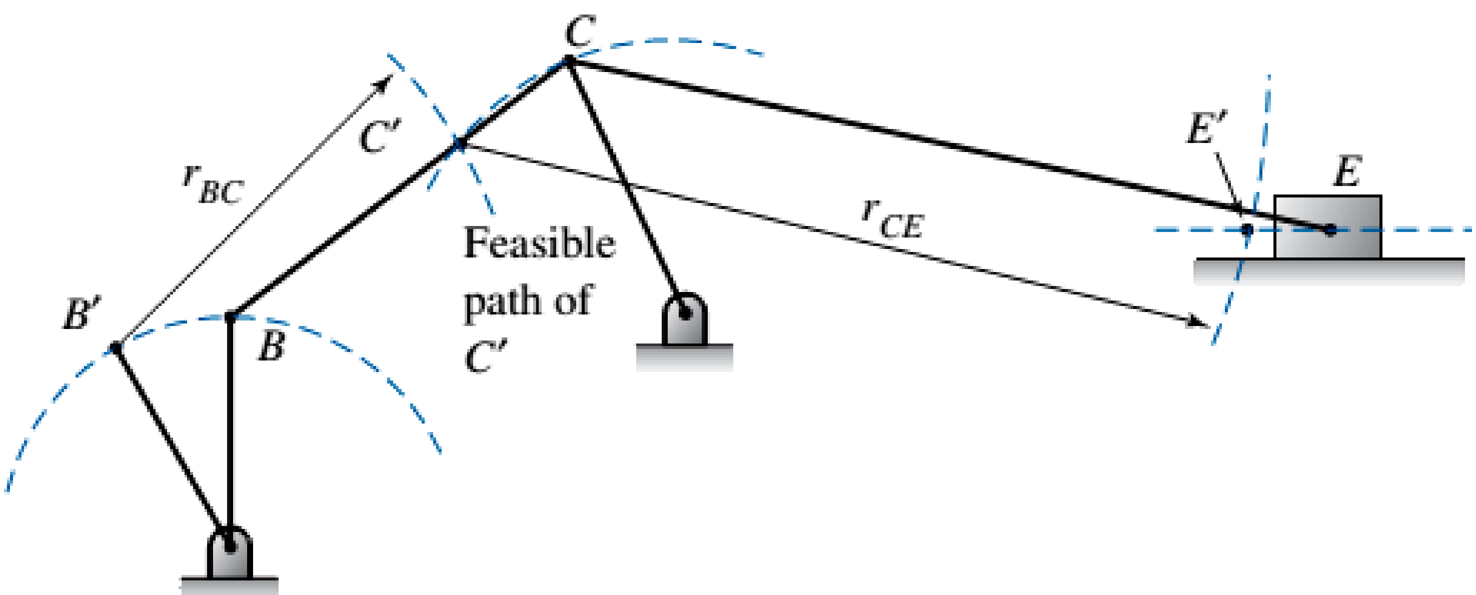

Feasible path of C′
Intersection represents
precise location of C′
r_{BC}
C
B′
B
3
Constrained
path of point C
15°
2
4
r_{CD}
D
A
r_{AB}
1
Second intersection represents
another geometric
inversion

EX. shows a kinematic diagram of a mechanism that is driven by moving link 2. Graphically reposition the links of the mechanism as link 2 is displaced 30° counterclockwise. Determine the resulting angular displacement of link 4 and the linear displacement of point .

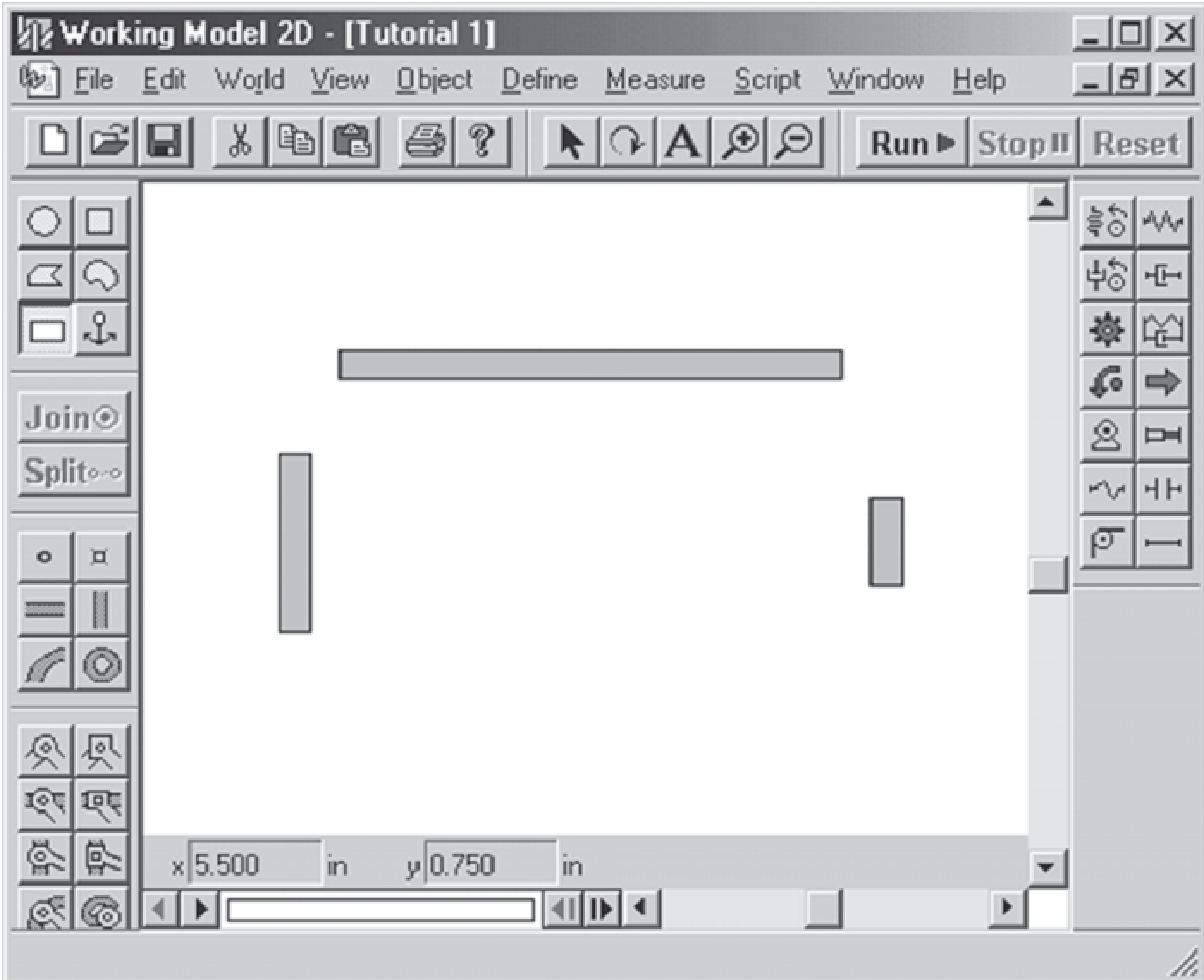


$$n = 6 \; j_p = (6 \text{ pins} + 1 \text{ sliding}) = 7 \; j_h = 0$$

$$M = 3(n-1) - 2j_p - j_h = 3(6-1) - 2(7) - 0 = 15 - 14 = 1$$

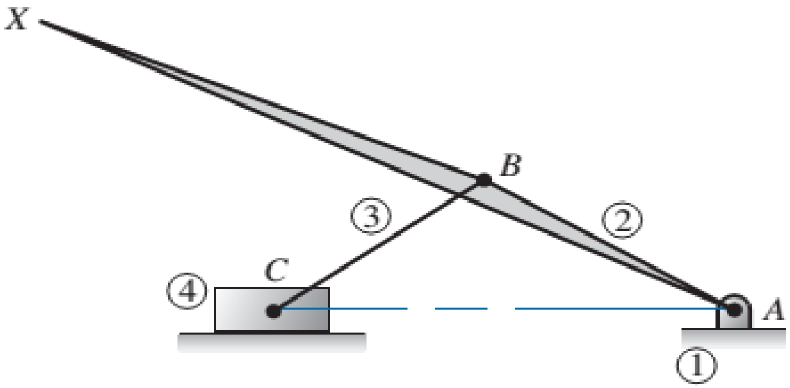


2.2 COMPUTER SIMULATION OF MECHANISMS

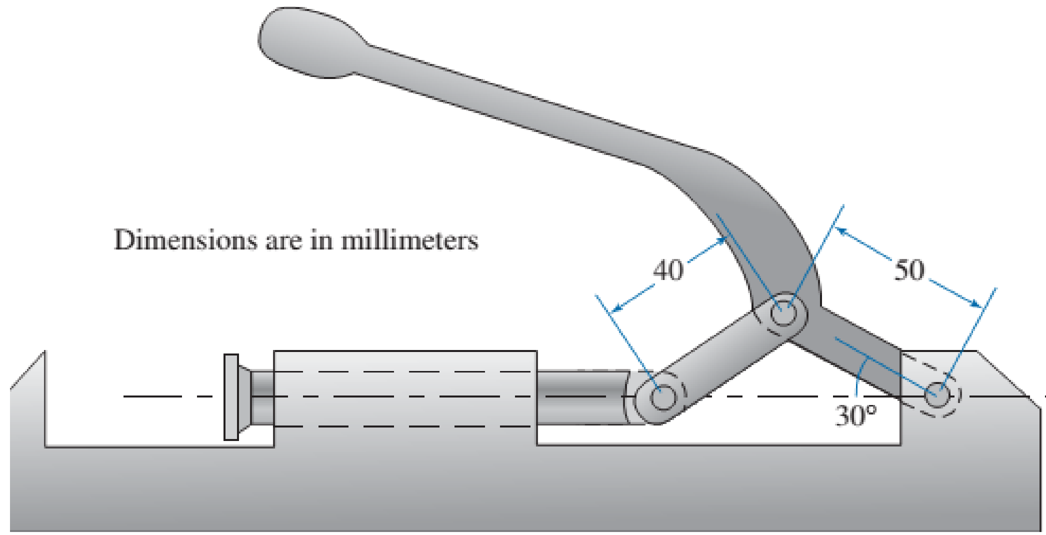


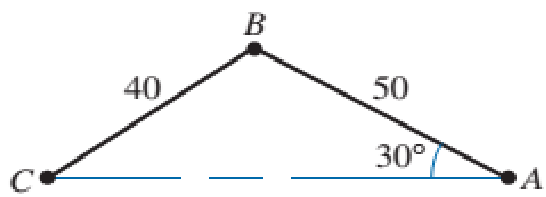
analysis
tomatic
ynamic
e com-
menus,
ponents
e links,
nts. The
onents,

Show a toggle clamp used to securely hold parts. Analytically determine the displacement of the clamp surface as the handle rotates downward, 15°.



(a)

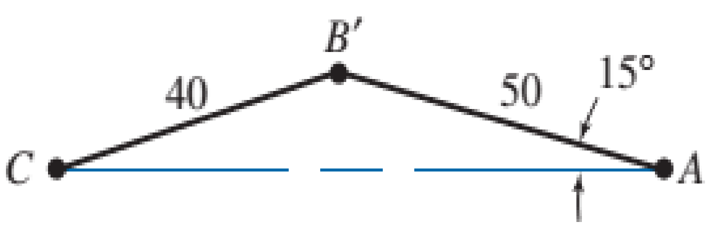




(b)

$$\frac{\sin\angle BAC}{(BC)} = \frac{\sin\angle BCA}{(AB)}$$

$$\angle BCA = \sin^{-1}\left[\left(\frac{AB}{BC}\right)\sin\angle BAC\right] = \sin^{-1}\left[\left(\frac{50\text{ mm}}{40\text{ mm}}\right)\sin 30^\circ\right] = 38.68^\circ$$



(c)

$$AC = \sqrt{AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC}$$

$$= \sqrt{(50\text{ mm})^2 + (40\text{ mm})^2 - 2(50\text{ mm})(40\text{ mm})\{\cos 111.32^\circ\}}$$

$$= 74.52\text{ mm}$$

$$\angle B'C'A = \sin^{-1}\left[\left(\frac{AB'}{B'C'}\right)\sin\angle C'AB'\right] = \sin^{-1}\left[\left(\frac{50\text{ mm}}{40\text{ mm}}\right)\sin 15^\circ\right] = 18.88^\circ$$

$$\angle AB'C' = 180^\circ - (15^\circ + 18.88^\circ) = 146.12^\circ$$

$$AC' = \sqrt{AB'^2 + B'C'^2 - 2(AB')(B'C')\cos\angle AB'C'}$$

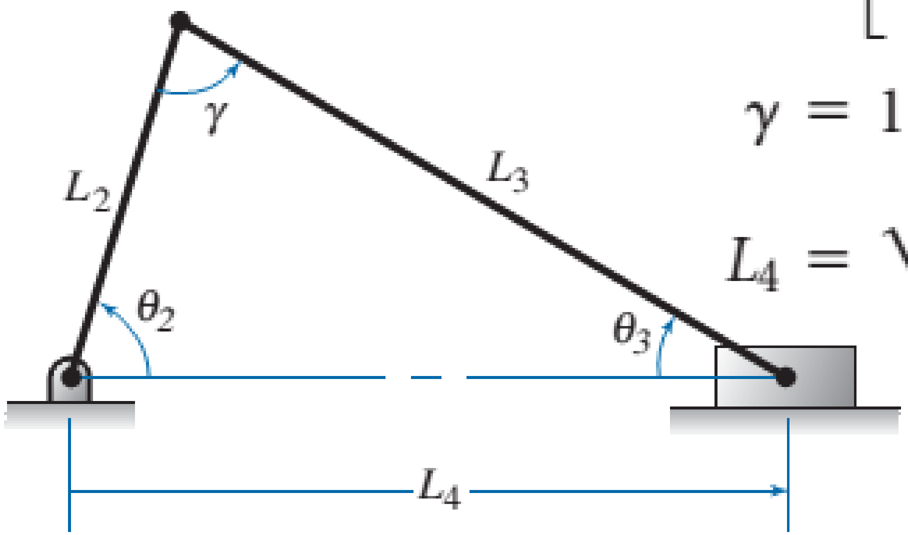
$$= \sqrt{(50\text{ mm})^2 + (40\text{ mm})^2 - 2(50\text{ mm})(40\text{ mm})\cos(146.12^\circ)} = 86.14\text{ mm}$$

$$= 86.14\text{ mm}$$

The displacement of point C during this motion can be found as the difference of the triangle sides AC' and AC:

$$\Delta R_C = AC' - AC = 86.14 - 74.52 = 11.62\text{ mm} \leftarrow$$

Closed-Form Position Analysis Equations for an In-Line Slider-Crank

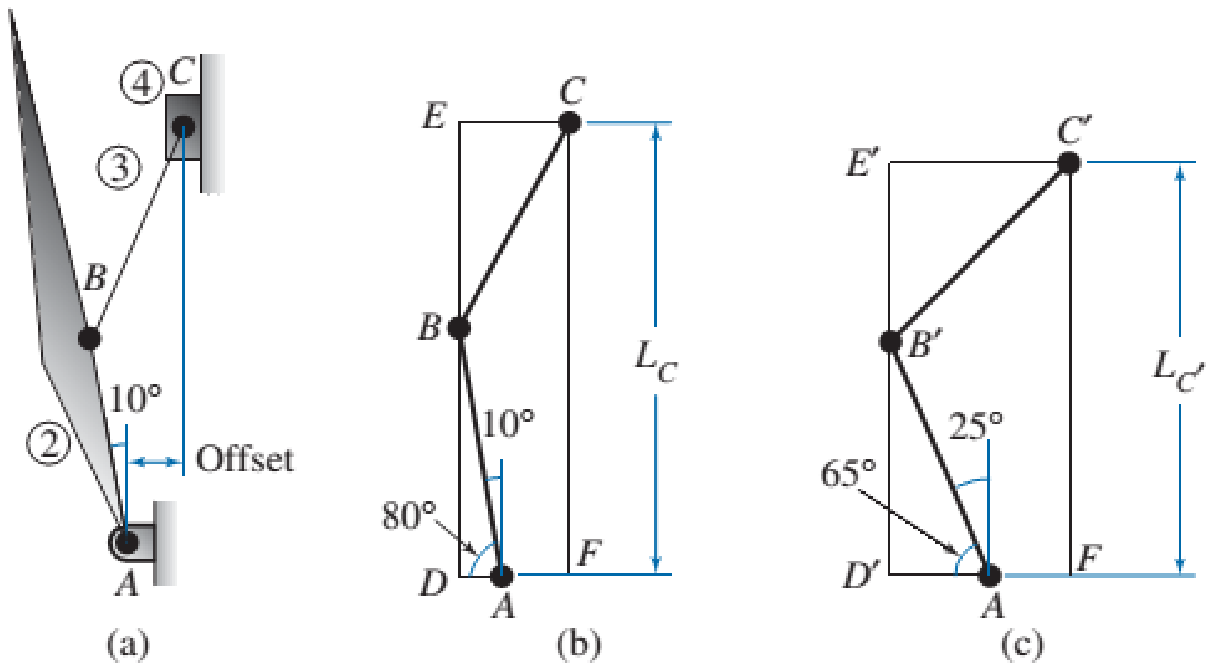


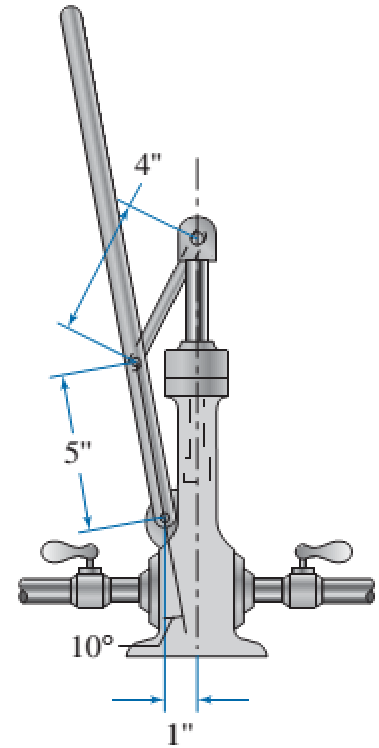
$$\theta_3 = \sin^{-1}\left[\frac{L_2}{L_3}\sin\theta_2\right]$$

$$\gamma = 180^\circ - (\theta_2 + \theta_3)$$

$$L_4 = \sqrt{L_2^2 + L_3^2 - 2(L_2)(L_3)\cos\gamma}$$

Fig shows a concept for a hand pump used for increasing oil pressure in a hydraulic line. Analytically determine the displacement of the piston as the handle rotates 15° counterclockwise.





$$\cos\angle BAD = \frac{AD}{AB}$$

$$AD = (AB)\cos\angle BAD = (5\text{in.})\ \{\cos 80°\} = 0.87\,\text{in.}$$

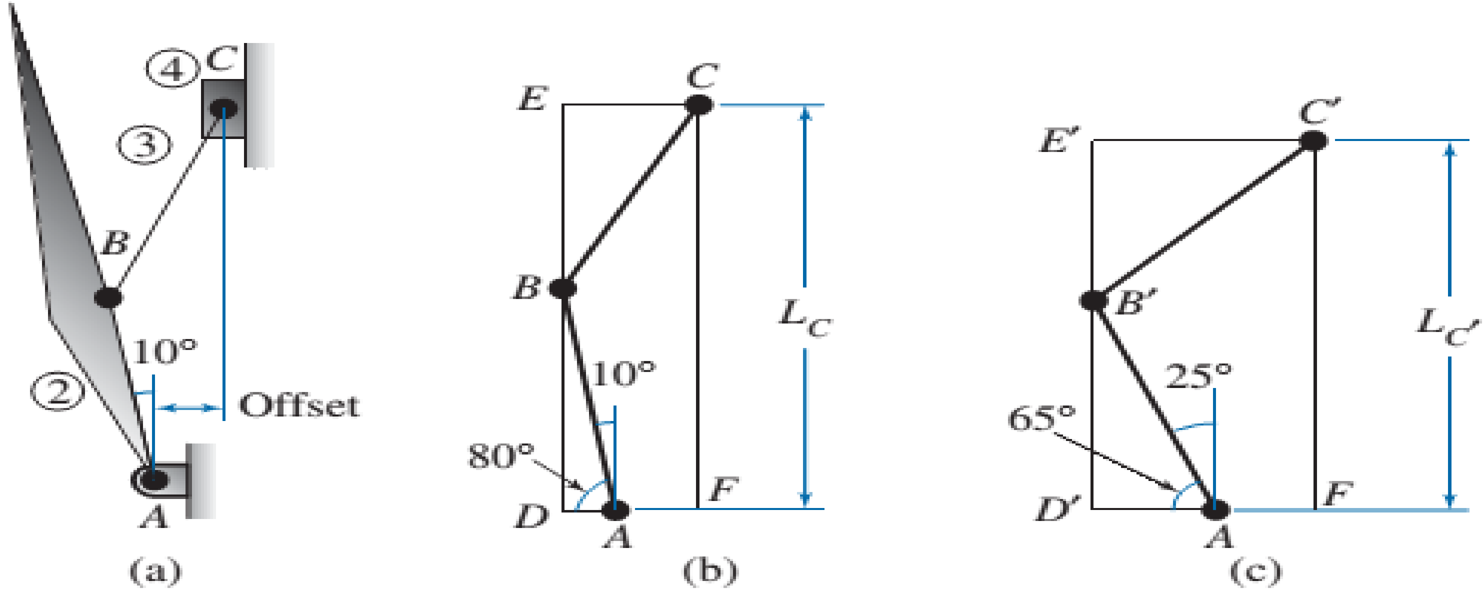
$$\sin\angle BAD = \frac{BD}{AB}$$

$$BD = (AB)\sin\angle BAD = (5\text{ in.})\ \{\sin 80°\} = 4.92\text{ in.}$$

$$CE = \text{offset} + AD = 1.0 + 0.87 = 1.87\,\text{in.}$$

Use the Pythagorean theorem, equation (3.4), to determine side *BE*:

$$BE = \sqrt{BC^2 - CE^2}$$
$$= \sqrt{(4)^2 - (1.87)^2} = 3.54\text{ in.}$$



$$\cos \angle BAD = \frac{AD}{AB}$$

$$AD = (AB) \cos \angle BAD = (5\,\text{in.}) \; \{\cos\ 80°\} = 0.87\,\text{in.}$$

$$\sin \angle BAD = \frac{BD}{AB}$$

$$BD = (AB) \sin \angle BAD = (5\ \text{in.}) \; \{\sin\ 80°\} = 4.92\ \text{in.}$$

$$CE = \text{offset} + AD = 1.0 + 0.87 = 1.87\,\text{in.}$$

Use the Pythagorean theorem, equation (3.4), to determine side *BE*:

$$BE = \sqrt{BC^2 - CE^2}$$

$$= \sqrt{(4)^2 - (1.87)^2} = 3.54\ \text{in.}$$

$$L_C = BD + BE = 4.92 + 3.54 = 8.46 \text{ in.}$$

Although not required in this problem, the angle that defines the orientation of link 3 is often desired. The angle $\angle BCE$ can be determined with the inverse cosine function:

$$\angle BCE = \cos^{-1}\left(\frac{CE}{BC}\right) = \cos^{-1}\left(\frac{1.87 \text{ in.}}{4 \text{ in.}}\right) = 62.13°$$

$$AD' = (AB')\cos\angle B'AD' = (5 \text{ in.})\{\cos 65°\} = 2.11 \text{ in.}$$

$$B'D' = (AB')\sin\angle B'AD' = (5 \text{ in.})\ \{\sin 65°\} = 4.53 \text{ in.}$$

$$\begin{aligned} C'E' &= AF + AD' \\ &= 1.0 + 2.11 = 3.11 \text{ in.} \end{aligned}$$

$$B'E' = \sqrt{(B'C')^2 - (C'E')^2} = \sqrt{(4 \text{ in.})^2 - (3.11 \text{ in.})^2} = 2.52 \text{ in.}$$

$$L'_C = B'D' + B'E' = 4.53 + 2.52 = 7.05 \text{ in.}$$

Calculate the Resulting Displacement $\Delta R_C = 8.46 - 7.05 = 1.41 \text{ in.} \downarrow$

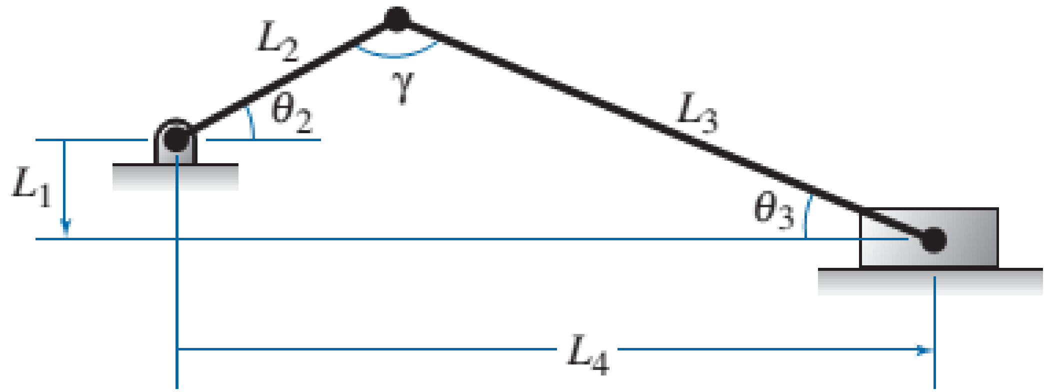


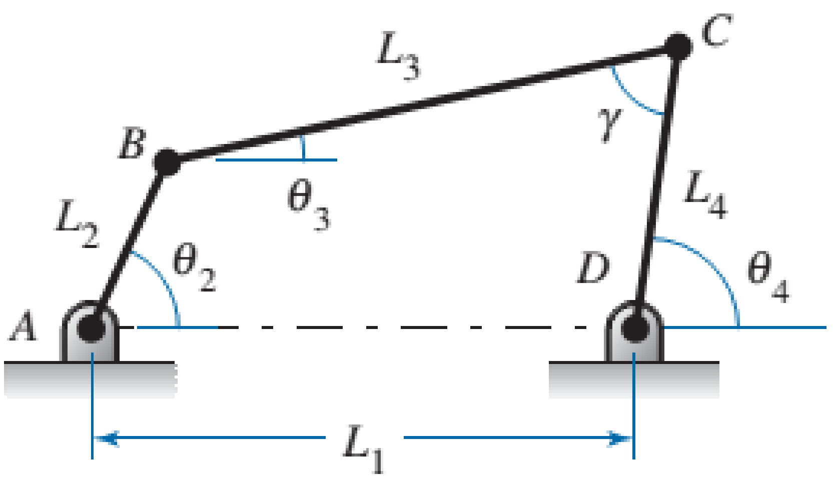
FIGURE 4.20 Offset slider-crank mechanism.

$$\theta_3 = \sin^{-1}\left[\frac{L_1 + L_2 \sin\ \theta_2}{L_3}\right]$$

$$L_4 = L_2 \cos \theta_2 + L_3 \cos \theta_3$$

$$\gamma = 180° - (\theta_2 + \theta_3)$$

Closed-Form Position Equations for a Four-Bar Linkage



$$BD = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos(\theta_2)}$$

$$\gamma = \cos^{-1}\left[\frac{(L_3)^2 + (L_4)^2 - (BD)^2}{2(L_3)(L_4)}\right]$$

$$\theta_3 = 2\tan^{-1}\left[\frac{-L_2 \sin\theta_2 + L_4 \sin\gamma}{L_1 + L_3 - L_2\cos\theta_2 - L_4\cos\gamma}\right]$$

$$\theta_4 = 2\tan^{-1}\left[\frac{L_2 \sin\theta_2 - L_3 \sin\gamma}{L_2\cos\theta_2 + L_4 - L_1 - L_3\cos\gamma}\right]$$

Circuits of a Four-Bar Linkage

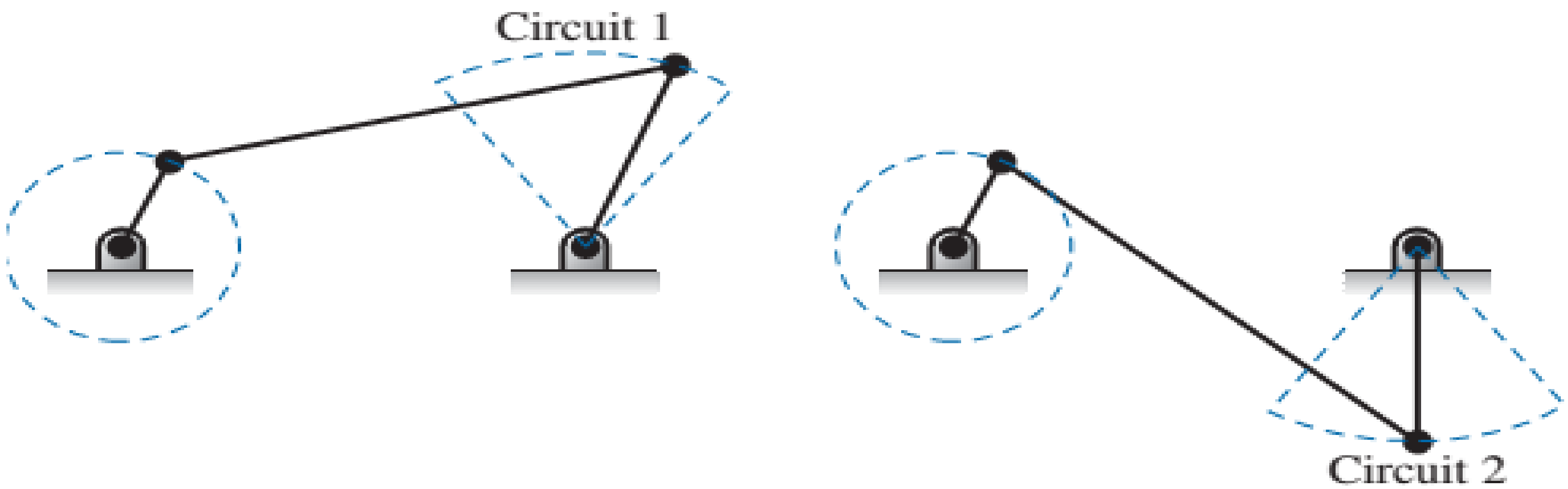


FIGURE 4.24 Circuits of a four-bar mechanism.

For four-bar mechanisms operating in the second circuit, equation (4.11) must be slightly altered as follows:

$$\theta_3 = 2\tan^{-1}\left[\frac{-L_2\sin\theta_2 - L_4\sin\gamma}{L_1 + L_3 - L_2\cos\theta_2 - L_4\cos\gamma}\right] \quad \textbf{(4.13)}$$

$$\theta_4 = 2\tan^{-1}\left[\frac{L_2\sin\theta_2 + L_3\sin\gamma}{L_2\cos\theta_2 + L_4 - L_1 - L_3\cos\gamma}\right] \quad \textbf{(4.14)}$$